Coherent Specifications of a Mixed Demand Model: The Stone—Geary Model

GianCarlo Moschini, *Iowa State University*

Pier Luigi Rizzi, *Universita' di Siena*

COHERENT SPECIFICATION OF A MIXED DEMAND SYSTEM: THE STONE-GEARY MODEL

by GianCarlo Moschini and Pier Luigi Rizzi*

THE SYSTEM-WIDE APPROACH TO DEMAND ANALYSIS has long been of interest to applied economists (Johnson, Hassan, and Green 1984). The most common specification of empirical models in this setting consists of expressing quantity demanded as a function of total expenditure and market prices—e.g., via a "direct" demand system. Whereas this approach corresponds to the usual representation of the individual consumer problem, its use within an econometric model requires some additional identifying assumptions—essentially, what is to be assumed as exogenous or predetermined. The standard specification of demand models with quantities consumed as the dependent variable thus relies on the implicit assumption that prices (and total expenditure) are predetermined. That is, if one thinks of the data at hand as the outcome of a market equilibrium model, the implicit assumption is that supply functions are perfectly elastic so that demands adjust to clear the market.

This condition may hold for aggregate (market) data in some situations—for example, when modeling the demand of tradeable goods for a small, open economy, or when prices are administratively set (e.g., public utilities). But often the implicit assumption that supplies are perfectly elastic is not tenable. Geary (1949–50) noted earlier on that "From the regression viewpoint, however, it would be equally logical to regard prices as dependent variables and quantities as independent variables…." This view, which is in keeping with the convention of representing demand curves with prices on the vertical axis, suggests an alternative assumption: quantities are predetermined, and prices adjust to clear the market along demand curves. In the system-wide approach to demand analysis, this leads to the specification of "inverse" demand systems, an approach that has been found particularly appealing when analyzing the demand for perishable products defined over a short period of time.

Direct or inverse demand systems have been the object of a great many applications. Examples of direct demand system specifications include the (direct) translog (Christensen, Jorgenson, and Lau 1975), the almost ideal demand system (Deaton and

* GianCarlo Moschini is Professor and the Pioneer Endowed Chair in Science and Technology Policy, Department of Economics, Iowa State University. Pier Luigi Rizzi is Professor, Dipartimento di Economia Politica, Universita' di Siena, Italy.

Muellbauer 1980b), the semiflexible almost ideal demand system (Moschini 1998), and the (direct) differential or Rotterdam model (Theil 1975 and 1976). Examples of inverse demand system specifications include the (inverse) translog (Christensen, Jorgenson, and Lau 1975), the (inverse) Rotterdam model (Theil 1975 and 1976, Barten and Bettendorf 1989), the linear inverse demand system (Moschini and Vissa 1992), and the normalized quadratic specification (Holt and Bishop 2002).

A third class of demand models is that of "mixed" demand functions, first introduced by Samuelson (1965): for some goods the prices are given, but for some others it is the quantity that is given, and prices adjust to clear the market. This class of models has obvious econometric appeal for the purpose of estimating demand behavior, because it encompasses the entire spectrum of possibilities between the polar cases of direct and inverse demand functions. Specifically, the use of mixed demands could allow for a much richer set of options about what is to be assumed as exogenous or predetermined in a demand system. Despite that, mixed demand functions have received comparatively little attention in applied studies. Heien (1977) invoked mixed demand theory to specify a reduced-form demand system for the meat sector, but theoretically consistent mixed demand systems have been scarce.

A feature that renders the specification of theory-consistent mixed demand functions challenging is that, in Samuelson's (1965) formulation, knowledge of both direct and indirect utility functions is required to characterize their properties. This means that many commonly used flexible functional forms, such as the translog or the almost ideal systems, cannot be used directly in this context because these flexible functional forms do not have a closed-form dual representation. Perhaps for this reason, the system-wide approach to mixed demands seems to have been confined to the differential approach. Barten (1992) appealed to mixed demand theory to illustrate the choice of which variables to assume as exogenous, but actually estimated a standard Rotterdam model (while taking into account the endogeneity of some of the prices in formulating the likelihood function). Moschini and Vissa (1993), by contrast, formulated and estimated a true differential mixed demand system, and this formulation was recently used by Matsuda (2004).[1]

Even though such a Rotterdam-type mixed demand system is of considerable interest, for some applications (such as welfare analysis) it may be desirable to have an exact parametric representation of preferences. In this chapter we revisit the question of how to specify an internally consistent system of mixed demand equations. We review the theoretical framework and show that mixed demand functions can be

[1] Mixed demands also turn up in environmental economics (e.g., Cunha-e-Sá and Ducla-Soares 1999), although the setting there is more appropriately one of rationed demands (the goods in fixed supply are public goods and do not enter the consumer's budget constraint).

explicitly derived for a special representation of preferences, namely the Stone-Geary utility function (Samuleson 1947–48, Geary 1949–50).

It is well known, of course, that Stone-Geary preferences are restrictive. The mixed demand system that we derive, however, presents several motives of interest. First, it offers a theory-consistent benchmark against which more flexible mixed demand specifications may be compared. Second, the Stone-Geary mixed demand system provides an appealing framework for exploring in more detail the econometric implications of choosing the "incorrect" set of predetermined variables in demand models. The question essentially boils down to which specification is affected by an error-in-variables problem, a situation that can lead to serious inconsistency in estimated parameters. To study this question one needs to compare direct and mixed (and inverse, if desirable) demand systems, and for that comparison to be meaningful the alternative specifications must all be traceable to the same underlying preference structure. Because, as we show, Stone-Geary preferences permit the explicit solution of both direct and mixed demand functions, the model that we present is rather useful for that purpose. We illustrate this with a simple Monte Carlo experiment that contrasts a six-good direct demand system with a comparable mixed demand system.

But the mixed demand system that we derive—which, to our knowledge, has not been presented elsewhere—should be of interest in its own right. The Stone-Geary utility representation of preferences played an important role in the early development of empirical demand analysis, leading to the development of the linear expenditure system (LES) (Klein and Rubin 1947–48, Stone 1954). The LES model has been used in countless applications, and its simplicity, exact aggregation properties, and parametric parsimony continue to make it of interest to practitioners. Its restrictive features include that it rules out inferior goods, and that it constrains the substitution possibilities across goods (all goods are net substitutes). Whether the latter is a serious problem depends on the intended application (in most cases, goods are in fact expected to be net substitutes). In any event, this feature allows the Stone-Geary mixed demand system that we derive to provide a nice illustration of Madden's (1991) *R*-classification (the generalization, to the case of rationed demands, of the notion of Hicksian substitutability and complementarity). Another restrictive feature of the LES system is that it entails linear Engel curves (the underlying utility function is a special case of quasi-homothetic preferences). This attribute is most damaging in the context of complete demand systems, where Engel curves are expected to display substantial nonlinearity (Deaton and Muellbauer 1980a). But in conditional demand systems that model only a weakly separable partition of the consumption bundle, such as the one we present in our application to vegetable demand in Italy, this restriction on the shape of Engel curves may be less important.

Mixed Demands

CONSIDER A CONSUMER WHO ALLOCATES disposable income to $(m+n)$ goods. Let $x \equiv (x_1, x_2, ..., x_n)$ denote the vector of commodities chosen optimally and let $z \equiv (z_1, z_2, ..., z_m)$ denote the vector of commodities in fixed quantity whose prices are optimally determined. Correspondingly, p_i denotes the nominal price of x_i, whereas q_k denotes the nominal price of z_k. Total consumer expenditure (income, for short) is y. Mixed demands can then be derived from the constrained optimization problem (Samuelson 1965):

$$\max_{x,q} \quad U(x,z) - V(p,q,y) \quad s.t. \quad px + qz = y\,, \tag{1}$$

where $U(.)$ and $V(.)$ are the direct and indirect utility functions, respectively, which are assumed quasiconcave and quasiconvex in their respective arguments, as well as satisfying standard monotonicity properties.[2] The solution to (1) gives Marshallian mixed demand vectors $x^{*}{=}x(p,z,y)$ and $q^{*}{=}q(p,z,y)$. Clearly, at the optimum, $U(x^{*},z){=}\ V(p,q^{*},y) \equiv V^{M}(p,z,y)$, where V^M is the mixed utility function.

The mixed demand functions $x(p,z,y)$ and $q(p,z,y)$ satisfy Walras's law (the adding-up condition), that is, $p \cdot x(p,z,y) + q(p,z,y) \cdot z = y$. Moreover, the functions $x(p,z,y)$ are homogeneous of degree zero in (p,y), whereas the functions $q(p,z,y)$ are homogeneous of degree one in (p,y). It follows also that the mixed utility function is homogeneous of degree zero in p and y. The symmetry property that applies to mixed demand functions can be characterized in terms of the restricted expenditure function $C(p,z,u)$ used in the related area of rationed demand (Gorman 1976, Neary and Roberts 1980):

$$C(p,z,u) \equiv \min_{x} \left\{ p \cdot x \,\middle|\, U(x,z) \geq u \right\}. \tag{2}$$

Although mixed demands and rationed demands share important similarities, they should be carefully distinguished (in the case of rationed demands some markets do not clear). But compensated mixed demands are the same as the compensated demands under rationing (Chavas 1984). The cost function $C(p,z,u)$ is nondecreasing in p, nonincreasing in z, increasing in u, and homogeneous of degree one and concave

[2] Samuelson (1965) and Chavas (1984) represent the indirect utility function in terms of normalized prices p_i/y, which helps the derivation of duality relations. As long as y is given, however, the representation in (1) is admissible and simplifies somewhat the interpretation of the model.

in p. Also, the restricted cost function $C(p,z,u)$ is convex in z if the utility function is quasiconcave (Deaton 1981).

From Shephard's lemma, the partial derivatives of $C(p,z,u)$ with respect to p give the compensated (i.e., Hicksian) mixed demands for goods whose quantity is optimally chosen, i.e., the solutions to problem (2), denoted $x^h(p,z,u)$. Moreover, the partial derivatives of $C(p,z,u)$ with respect to z give (the negative of) the vector of compensated shadow or virtual prices of the goods in predetermined quantity. These shadow prices, denoted $q^h(p,z,u)$, are the compensated price-dependent demand functions of z, that is, the prices that would have resulted in the quantity z actually being the cost-minimizing solution. Specifically,

$$\nabla_p C(p,z,u) = x^h(p,z,u) \tag{3}$$

$$\nabla_z C(p,z,u) = -q^h(p,z,u) \,. \tag{4}$$

The compensated demand functions $x^h(p,z,u)$ are homogeneous of degree zero in p whereas the compensated price functions $q^h(p,z,u)$ are homogeneous of degree one in p. Curvature and symmetry conditions imply that the matrix of partial derivatives $\nabla_p x^h(p,z,u)$ is symmetric and negative semi-definite, that the matrix of partial derivatives $\nabla_z q^h(p,z,u)$ is symmetric and negative semi-definite, and that $\nabla_z x^h(p,z,u) = -\nabla_p q^h(p,z,u)$. These conditions also imply that the Hessian of the restricted cost function is skew symmetric.

The Hicksian mixed demand functions in (3) and (4) are related to the Marshallian mixed demand functions that solve problem (1) by the standard identities:

$$x(p,z,y) = x^h\left(p,z,V^M(p,z,y)\right) \tag{5}$$

$$q(p,z,y) = q^h\left(p,z,V^M(p,z,y)\right), \tag{6}$$

where $V^M(p,z,y)$ is the "mixed utility function" defined earlier. In principle, the mixed utility may be obtained from the restricted cost function by solving for u the identity

$$C\left(p,z,u\right) - \nabla_z C\left(p,z,u\right)\cdot z \equiv y \,. \tag{7}$$

Clearly, it is not possible to obtain a closed-form solution for $V^M(p,z,y)$ for an arbitrary specification of $C(p,z,u)$. For this reason, for example, the PIGLOG cost function used by Deaton (1981) to model rationed demand is not particularly appealing in the present context. Elsewhere (Moschini and Rizzi 2006) we show that an appropriate choice for the parameterization of $C(p,z,u)$ can provide a mixed demand system that satisfies the standard requirements of a "flexible functional form." In what follows, however, we follow the direct route of solving the problem in (1) for a specific representation of preferences for which we can write both direct and indirect utility functions in closed form, namely the case of Stone-Geary preferences.

The Stone-Geary Mixed Demand System

THE STONE-GEARY UTILITY FUNCTION played an important role in the early development of empirical demand analysis. The LES derived by Klein and Rubin (1947-48), later systematically implemented by Stone (1954) in the first large-scale demand system estimation, was shown to be integrable into a simple direct utility function by Geary (1949-50) and Samuleson (1947-48). A concise and informative background discussion is provided by Neary (1997), who points out that Nash's (1953) solution to his classic axiomatic bargaining problem also entails a function of the Stone-Geary type. The direct and indirect utility functions for Stone-Geary preferences are written as

$$U(x,z)=\prod_{i=1}^{n}(x_i-\gamma_i)^{\alpha_i}\prod_{k=1}^{m}(z_k-\delta_k)^{\beta_k} \tag{8}$$

$$V(p,q,y)=\mu\left(y-\sum_{i=1}^{n}p_i\gamma_i-\sum_{k=1}^{m}q_k\delta_k\right)\prod_{i=1}^{n}p_i^{-\alpha_i}\prod_{k=1}^{m}q_k^{-\beta_k}\ , \tag{9}$$

where $\alpha_i>0, \forall i$, and $\beta_k>0, \forall k$, and $\mu=\prod_{i=1}^{n}\alpha_i^{\alpha_i}\prod_{k=1}^{m}\beta_k^{\beta_k}$. Regularity conditions also require $(x_i-\gamma_i)>0,\ \forall i$, $(z_s-\delta_s)>0,\ \forall s$, and without loss of generality we put $\sum_{i=1}^{n}\alpha_i+\sum_{k=1}^{m}\beta_k=1$.

Given this parameterization of preferences, the first-order conditions for the problem in (1) are

$$\frac{\alpha_j}{x_j-\gamma_j}=\lambda p_j, \quad j=1,\ldots,n \tag{10}$$

$$\frac{\delta_s}{\left(y-\sum_{i=1}^{n} p_i\gamma_i - \sum_{k=1}^{m} q_k\delta_k\right)} + \frac{\beta_s}{q_s} = \lambda z_s, \quad s=1,\ldots,m \tag{11}$$

$$\sum_{i=1}^{n} p_i x_i + \sum_{k=1}^{m} q_k z_k = y, \tag{12}$$

where λ is the Lagrange multiplier. The solution to this set of equations, slightly more involved than those of the standard Stone-Geary direct utility maximization problem (see the Appendix), yields the following mixed demand equations:

$$x_j^* = \gamma_j + \frac{\alpha_j}{p_j}\frac{\left(y-\sum_{i=1}^{n} p_i\gamma_i\right)}{\left(\sum_{k=1}^{m}\frac{z_k\beta_k}{z_k-\delta_k}+\sum_{i=1}^{n}\alpha_i\right)}, \quad j=1,2,\ldots,n \tag{13}$$

$$q_s^* = \left(\frac{\beta_s}{z_s-\delta_s}\right)\frac{\left(y-\sum_{i=1}^{n} p_i\gamma_i\right)}{\left(\sum_{k=1}^{m}\frac{z_k\beta_k}{z_k-\delta_k}+\sum_{i=1}^{n}\alpha_i\right)}, \quad s=1,\ldots,m. \tag{14}$$

The Hicksian mixed demand functions are obtained by solving the problem in (2) given the utility function in (8), yielding

$$x_j^h = \gamma_j + \frac{\alpha_j}{p_j}u^{1/a}\left[\prod_{i=1}^{n}\left(\frac{p_i}{\alpha_i}\right)^{\alpha_i}\right]^{1/a}\left[\prod_{k=1}^{m}(z_k-\delta_k)^{\beta_k}\right]^{-1/a}, \quad j=1,2,\ldots,n \tag{15}$$

and the associated restricted (rationed) cost function

$$C(p,z,u) = \sum_{j=1}^{n} p_j\gamma_j + a\,u^{1/a}\left[\prod_{i=1}^{n}\left(\frac{p_i}{\alpha_i}\right)^{\alpha_i}\right]^{1/a}\left[\prod_{k=1}^{m}(z_k-\delta_k)^{\beta_k}\right]^{-1/a}, \tag{16}$$

where $a \equiv \sum_{i=1}^{n}\alpha_i$. Differentiating this cost function yields the compensated shadow price equations $q_s^h \equiv -\partial C(p,z,u)/\partial z_s$; that is,

$$q_s^h = \frac{\beta_s}{z_s - \delta_s} u^{1/a} \left[\prod_{i=1}^{n} \left(\frac{p_i}{\alpha_i} \right)^{\alpha_i} \right]^{1/a} \left[\prod_{k=1}^{m} (z_k - \delta_k)^{\beta_k} \right]^{-1/a}, \quad s = 1, \ldots, m. \tag{17}$$

The Hicksian mixed demands of the Stone-Geary parameterization permit a nice illustration of Madden's (1991) R-classification, that is, the generalization, to the case of rationed demands, of the notion of Hicksian substitutability and complementarity. Differentiating the Hicksian mixed demand and shadow price equations in (15) and (17), it is easily verified that

$$\frac{\partial x_j^h}{\partial p_\ell} > 0, \quad \forall j, \ell = 1, \ldots, n, \quad j \neq \ell$$

$$\frac{\partial x_j^h}{\partial z_s} < 0, \quad \forall j = 1, \ldots, n, \quad s = 1, \ldots, m$$

$$\frac{\partial q_s^h}{\partial z_r} < 0, \quad \forall s, r = 1, \ldots, m, \quad s \neq r$$

$$\frac{\partial q_s^h}{\partial p_j} > 0, \quad \forall s = 1, \ldots, m, \quad j = 1, \ldots, n.$$

Thus, for the Stone-Geary mixed demand system, we find that all goods behave in accordance with Madden's (1991) definition of R-substitutes. This should come as no surprise because it is well know that the Stone-Geary preferences, in the context of ordinary demand functions, force all goods to behave as Hicksian substitutes.

Direct or Mixed Demands? A Monte Carlo Illustration

BECAUSE THE STONE-GEARY PREFERENCES allow for explicit and internally consistent derivation of both direct demands and mixed demands, they offer an interesting opportunity to explore the consequences of choosing which variables to assume as predetermined in applied demand analysis. Here we illustrate with a simple Monte Carlo example. We will compare two specifications: (a) all expenditure-deflated prices are predetermined; and (b) for one-half of the goods the prices are predetermined and for the other half it is the quantities that are predetermined. Thus, (a) leads to the standard direct demand system framework, whereas (b) leads to the mixed demand system framework.

From the postulated Stone-Geary utility function, the direct demand equations are written as

$$x_j^* = \gamma_j + \frac{\alpha_j}{p_j}\left(y - \sum_{i=1}^{n} \gamma_i p_i - \sum_{k=1}^{m} \delta_k q_k \right), \quad j = 1,2,\dots,n \tag{18}$$

$$z_s^* = \delta_s + \frac{\beta_s}{q_s}\left(y - \sum_{i=1}^{n} \gamma_i p_i - \sum_{k=1}^{m} \delta_k q_k \right), \quad s = 1,2,\dots,m. \tag{19}$$

As in many demand systems specifications, we will work with budget shares as the dependent variable, where the shares are defined as $w_i \equiv p_i x_i / y$ and $w_s \equiv q_s z_s / y$. Hence, direct and mixed demand systems have the same observable left-hand-side variable, which facilitates comparison. For the direct demand functions in (18)–(19) we have

$$w_j^* = \gamma_j \frac{p_j}{y} + \alpha_j \left(1 - \sum_{i=1}^{n} \gamma_i \frac{p_i}{y} - \sum_{k=1}^{m} \delta_k \frac{q_k}{y} \right), \quad j = 1,2,\dots,n \tag{20}$$

$$w_s^* = \delta_s \frac{q_s}{y} + \beta_s \left(1 - \sum_{i=1}^{n} \gamma_i \frac{p_i}{y} - \sum_{k=1}^{m} \delta_k \frac{q_k}{y} \right), \quad s = 1,2,\dots,m. \tag{21}$$

For the mixed demand equations in (13)–(14), on the other hand, we have

$$w_j^* = \gamma_j \frac{p_j}{y} + \alpha_j \frac{\left(1 - \sum_{i=1}^{n} \frac{p_i}{y} \gamma_i \right)}{\left(\sum_{k=1}^{m} \frac{z_k \beta_k}{z_k - \delta_k} + \sum_{i=1}^{n} \alpha_i \right)}, \quad j = 1,2,\dots,n \tag{22}$$

$$w_s^* = \left(\frac{z_s \beta_s}{z_s - \delta_s} \right) \frac{\left(1 - \sum_{i=1}^{n} \frac{p_i}{y} \gamma_i \right)}{\left(\sum_{k=1}^{m} \frac{z_k \beta_k}{z_k - \delta_k} + \sum_{i=1}^{n} \alpha_i \right)}, \quad s = 1,\dots,m. \tag{23}$$

In this Monte Carlo experiment we consider the case of six goods and set $n = 3$ and $m = 3$. The postulated true preference parameters, held fixed throughout, are reported in Table 1. For ease of interpretation, this table also reports the own-price elasticities and the expenditure elasticities that are implied by these parameters (at the mean point of the data generated).

Table 1. Stone-Geary Parameters of True Model

Parameters				Direct Demand Elasticities		
				Good	Own-price	Expenditure
α_1	0.08	γ_1	0.04	x_1	-0.69	0.67
α_2	0.18	γ_2	0.07	x_2	-0.77	0.72
α_3	0.24	γ_3	-0.06	x_3	-1.25	1.33
β_1	0.10	δ_1	0.08	z_1	-0.60	0.56
β_2	0.15	δ_2	-0.05	z_2	-1.43	1.50
β_3	0.25	δ_3	-0.08	z_3	-1.35	1.47

elasticities and the expenditure elasticities that are implied by these parameters (at the mean point of the data generated).

The experiment proceeds under the alternative assumptions that the structural part of the true model is either the system of direct demands in (20)–(21) or the system of mixed demands in (22)–(23). Consider first the case when the true data-generating process (DGP) is that of direct demand functions. In such a case, for each replication, we proceed as follows:

(a) Fix the sample size T and the parameters' true value (as given in Table 1).

(b) Generate the right-hand-side variables of the share equations in (20)–(21). Specifically, we put $y = 1$ throughout (so that prices can be interpreted as expenditure-deflated prices, as suggested by the homogeneity property) and generate (by using a pseudo-random number generator) "true" prices $\overline{p}_i$ and $\overline{q}_s$ as normal variables with mean one and with covariance $\Sigma \equiv [\sigma_{\ell r}]$, where $\sigma_{\ell\ell} = 0.20$, $(\forall \ell = 1,...,m+n)$, and $\sigma_{\ell r} = 0.6 \times (\sigma_{\ell\ell})^2$, $(\forall \ell \neq r = 1,...,m+n)$.

(c) Generate the "true" quantity values, denoted $\overline{x}_i$ and $\overline{z}_s$, by using (18)–(20) and the prices $\overline{p}_i$ and $\overline{q}_s$ generated in the previous step.

(d) Generate the vectors or error terms, by using a pseudo-random number generator, as zero-mean normal variates $e \sim N(0,\Omega)$, where the covariance matrix has the following structure: $\Omega \equiv [\Omega_{\ell r}]$, $\ell, r = 1,...,m+n-1$, where $\Omega_{\ell\ell} = \omega_\ell^2$, $\Omega_{\ell r} = \rho\omega_\ell\omega_r$ $(\forall \ell \neq r)$, $\omega_\ell = 0.01$ $(\forall \ell)$, and $\rho = -0.2$.

(e) Generate the "observed" prices as $p_i = \overline{p}_i$ and $q_k = \overline{q}_k$, and the "observed" quantities as $x_i = \overline{x}_i + (y/\overline{p}_i)e_i$ and $z_k = \overline{z}_k + (y/\overline{q}_k)e_k$. Compute "observed" shares as $w_i = p_i x_i / y$ and $w_k = q_k z_k / y$. Note that, by construction, $w_i = \bar{w}_i + e_i$ and $w_k = \bar{w}_k + e_k$, where $\bar{w}_i \equiv \bar{p}_i \bar{x}_i / y$ and $\bar{w}_k \equiv \bar{q}_k \bar{z}_k / y$.

(f) Estimate the parameters of the Stone-Geary model by the standard maximum likelihood estimator for seemingly unrelated regressions (as discussed below in the context of the empirical application) for both the direct demand share system in (20)–(21) (the true model in this case) and the mixed demand system in (22)–(23) (the misspecified model in this case).

The foregoing procedure is readily modified for the case when the mixed demand system is the true model (and thus the direct demand system is the misspecified model). In that case, step (e) is replaced by the following:

(e′) Generate the "observed" prices as $p_i = \bar{p}_i$ and $q_k = \bar{q}_k + (y/\bar{z}_k)e_k$, and the "observed" quantities as $x_i = \bar{x}_i + (y/\bar{p}_i)e_i$ and $z_k = \bar{z}_k$. Compute "observed" shares as $w_i = p_i x_i / y$ and $w_k = q_k z_k / y$. Note that, by construction, $w_i = \bar{w}_i + e_i$ and $w_k = \bar{w}_k + e_k$, where $\bar{w}_i \equiv \bar{p}_i \bar{x}_i / y$ and $\bar{w}_k \equiv \bar{q}_k \bar{z}_k / y$.

To elaborate briefly on this Monte Carlo experiment, note that in step (b) we have assumed some positive correlation between expenditure-deflated prices (as typically found in applications of demand analysis). By contrast, the covariance structure in step (d) maintains a negative correlation between share errors (as naturally occurs because the budget constraint require errors to add up to zero over all equations). Also relevant is the signal-to-noise ratio that is implied by the covariance structure of step (d). That is best illustrated by what it implies in terms of fit for the estimated equations. If R_ℓ^2 denotes the standard measure of fit for equation $\ell = 1,2,...,m+n$, on average (as calculated in the experiment presented below) the parameterization that we have chosen entails the following fit for the true models (either direct or mixed): $R_1^2 = 0.40$, $R_2^2 = 0.65$, $R_3^2 = 0.58$, $R_4^2 = 0.70$, $R_5^2 = 0.50$, and $R_6^2 = 0.70$.

The experiment just described, with sample size $T = 25$, was replicated $N = 2{,}000$ times, and some results are reported in Table 2. Specifically, in this table, and for each model, we report the average percent bias for each parameter. For example, for parameter α_i the average percent bias is computed as

$$\frac{1}{N}\sum_{r=1}^{N}\left(\frac{\hat{\alpha}_i^r - \alpha_i}{\alpha_i}\right)\times 100,$$

where $\hat{\alpha}_i^r$ is the estimated parameter in the rth replication, and α_i is the corresponding true parameter value (as in Table 1). Similar definitions apply to the other para-

Table 2. Average Percent Bias of Estimated Parameters

	True DGP: Direct Demand System		True DGP: Mixed Demand System	
Parameter	Mixed Model	Direct Model	Mixed Model	Direct Model
α_1	-7.06	-0.17	-0.52	7.30
α_2	-6.08	0.06	-0.35	5.93
α_3	-5.36	0.13	-0.23	5.10
β_1	21.12	-0.06	0.13	-1.72
β_2	6.26	-0.16	0.77	-14.02
β_3	-0.42	0.01	0.12	-2.40
γ_1	-11.33	0.62	0.36	15.53
γ_2	-17.32	0.22	0.03	23.03
γ_3	-29.99	0.23	-0.30	38.78
δ_1	-47.11	0.16	-0.56	19.50
δ_2	-62.27	0.98	-4.02	78.66
δ_3	-41.07	0.44	-1.71	50.45
Mean absolute value	21.28	0.27	0.76	21.87

Note: Number of replications: N = 2,000; sample size: T = 25.

meters. It is clear that estimated true models display essentially no bias. The average (over all parameters) absolute percent bias is 0.27 for the direct model when it corresponds to the true DGP, and it is 0.76 for the mixed model when that is the true model. On the other hand, estimating the mixed model when the true DGP is the direct model, or estimating the direct model when the true DGP is the mixed model, entails considerable bias. The average (over all parameters) absolute percent bias in such cases is 21.87 for the direct model and 21.28 for the mixed model. These conclusions are supported by the average percent root mean square errors (RMSE) for each parameter (not reported here), which account for the sampling variance of the estimators (in addition to the bias). It is clear that the performance of the direct or mixed demand models depends on whether or not they correspond to the true DGP.

Finally, Table 3 illustrates the finite-sample properties of the estimators considered as the sample size increases. Specifically, to get an idea of the asymptotic convergence, we allow the sample size to increase from 25 to 400. Increasing the sample size does not help the precision of the estimates if the assumptions about which variables are the predetermined ones do not correspond to the true DGP. Estimating the "incorrect" model clearly leads to inconsistent parameter estimates. This, of course, is to be fully expected. Estimating the direct model when in fact the mixed demand

Table 3. Average (Absolute) Percent Bias and Sample Size

		Sample size				
True model	Estimating Model	25	50	100	200	400
Direct	Direct	0.27	0.20	0.09	0.08	0.07
	Mixed	21.28	20.76	20.33	20.27	20.15
Mixed	Direct	21.87	21.91	21.64	21.76	21.76
	Mixed	0.76	0.53	0.22	0.15	0.09

model is the correct specification essentially entails an error-in-variable problem. In such a situation, standard estimation techniques lead to inconsistent estimators (Fuller 1987). In particular, as in Moschini (2001), standard seemingly unrelated regression estimation of demand systems is bound to give inconsistent estimates of the underlying preference parameters.

An Application to the Italian Demand for Vegetables

THE STONE-GEARY MIXED DEMAND SYSTEM derived here is illustrated with an application to the demand for vegetables using Italian monthly. These data comprise both fresh vegetables as well as canned and frozen vegetables. Our presumption here is that, in this setting, the supplies of fresh vegetables are predetermined and perishable, so that prices adjust to clear the market for these goods. On the other hand, for frozen vegetables and canned vegetables (which are easily stored), the standard presumption that prices are given, and quantities adjust, seems more acceptable. Thus, as in Barten's (1992) application, this data set appears to fulfill the basic assumptions underlying the applicability of a mixed demand system. Of course, to apply our mixed demand system to this subset of goods at the aggregate level, we also need to postulate that the representative consumer's preferences are weakly separable in the appropriate partition (Blackorby, Primont, and Russell 1978).

Data

The data used in this study were obtained courtesy of the Italian statistical institute ISMEA (Istituto di Servizi per il Mercato Agricolo Alimentare). As part of their monitoring efforts on food consumption patterns, ISMEA maintains an extensive household data collection system (the "*Panel Famiglie*") in partnership with ACNielsen. This effort is based on records of purchases made by a sample of 6,000 Italian households. The sample was meant to be representative, stratified according to socio-demographic and location variables. Data of interest are recorded through the “home scanning” technology: every household in the sample is provided with a computer equipped with an optical scanner, which is used to record consumption information as soon as the purchased product enters the home. Data recorded in this fashion is supplemented by additional information concerning the purchase, through a computer-guided questionnaire, and a procedure exists to record comparable information for items without a bar code. Such elementary purchase data are electronically retrieved from each household on a weekly basis and are then aggregated for four-week intervals.

The data used here, extracted from that databank, concern consumption of vegetables aggregated at the Italian national level for the period January 1997 to April 2004. For each individual vegetable product we observe the total expenditure, the quantity, and the price (i.e., the unit cost, measured in euro/kg). The original 95 four-week observations were reduced to 88 monthly observations by averaging observations provided in the same calendar month (August 1997 and 1998, July 1999 and 2000, June 2001 and 2002, and May 2003). The long list of individual items was aggregated into the following nine products: (1) tomatoes, (2) eggplants, (3) zucchini, (4) bell peppers, (5) lettuce (including chicory and radicchio), (6) other vegetables (including fennel, carrots, asparagus, broccoli, artichokes, cauliflowers, cucumbers, onions, spinach, cabbage), (7) legumes (beans, green beans, and peas), (8) frozen vegetables, and (9) canned vegetables. Table 4 reports some descriptive statistics for these data. It is apparent that prices, and more so the quantities of fresh vegetables, are affected by strong fluctuations due to seasonal variations in both demand and supply.

Results

For the purpose of estimation we represent the mixed demand system in share form, as in (22)–(23). The conditional mixed demand system to be estimated is composed of nine equations, seven of which have the form of equation (23) and two of which have the form of equation (22). Prior to estimation, quantities z_k and prices p_i were normalized so as to have mean one. Given that $\sum_{i=1}^{n} w_i^* + \sum_{k=1}^{m} w_k^* = 1$, this constitutes a singular system, and therefore one equation is dropped prior to estimation.

The stochastic form of the estimating system can be represented as

$$w_t = f(X_t, \theta) + e_t\ , \quad t = 1, \ldots, 88\ , \tag{24}$$

where $w_t = (w_{1t}, \cdots, w_{8t})^T$ is the vector of $(m+n-1)$ budget shares for observation t; $f(\cdot)$ is the vector-valued function as per the structure in (22) and (23), with X_t representing the vector of all explanatory variables for observation t and θ denoting the vector of all parameters to be estimated; and $e_t = (e_{1t}, \cdots e_{8t})^T$ is the error vector.

Table 4. Descriptive Statistics of Data

	Prices (euro/kg)			Total Consumption (tons)		
Product:	Mean	Min.	Max.	Mean	Min.	Max.
1. Tomatoes	1.56	0.80	2.37	23,792	8,821	49,716
2. Eggplants	1.32	0.72	2.69	7,162	1,984	14,587
3. Zucchini	1.60	0.96	3.44	9,609	3,378	17,117
4. Bell peppers	1.74	1.08	2.56	7,385	3,369	15,559
5. Lettuce	1.62	1.25	2.73	19,798	10,739	26,202
6. Other vegetables	1.31	1.10	1.79	65,679	19,704	99,933
7. Legumes	2.11	1.40	3.06	5,583	1,016	14,706
8. Frozen vegetables	3.20	2.84	3.67	13,336	5,940	22,144
9. Canned vegetables	0.90	0.71	1.30	67,742	35,790	85,589
Vegetables total expenditure (1,000 euros)				298,697	190,248	359,791

Assuming that this error vector is multinormally distributed with zero mean and a constant contemporaneous covariance matrix allows maximum likelihood (ML) estimation (e.g., Davidson and MacKinnon 1993, chapter 9), which is invariant with respect to which equation is omitted. Preliminary analysis, however, suggested the need to consider two additional issues: seasonality, and serial correlation in the estimated residuals. To account for seasonality we allow the intercept-like parameters γ_j and δ_k to depend on seasonal quarterly dummy variables D_{dt}, where $d = 2,3,4$ indexes the quarter of each monthly observation not in the first quarter (these dummy variables were actually rescaled, by subtracting the own mean, to have a mean value of zero over the sample). Specifically,

$$\gamma_j = \gamma_{j1} + \sum_{d=2}^{4} \gamma_{jd} D_{dt} , \quad j = 1,2 \tag{25}$$

$$\delta_k = \delta_{k1} + \sum_{d=2}^{4} \delta_{kd} D_{dt} , \quad k = 1,...,7 . \tag{26}$$

As for autocorrelation in the errors, we allow for first-order serial correlation by estimating the model:

$$w_t = f(X_t, \theta) + R\left[w_{t-1} - f(X_{t-1}, \theta)\right] + \varepsilon_t , \quad t = 2,...,88 , \tag{27}$$

where $R = \rho I$, and where now the parameter vector θ includes the seasonal parameters as well. The model entails a total of 42 parameters (17 Stone-Geary coefficients, 24 seasonal parameters, and the coefficient of autocorrelation). Note that the autocorrelation coefficient is constrained to be the same for all equations, which provides the simplest structure guaranteeing that the resulting stochastic system satisfies adding-up (Berndt and Savin 1975, Moschini and Moro 1994). The standard assumptions leading to ML estimation, discussed earlier, now apply to the vector ε_t.

ML estimation of this system was carried out by using the software package TSP (version 4.5). The estimated parameters are reported in Table 5, except for the parameters of the seasonal effects, which are omitted (but because of our having scaled the seasonal dummy variables to have mean zero, the parameters γ_{j1} and δ_{k1} retain the same interpretation as the original parameters γ_j and δ_k).

The standard errors suggest that the parameters are all estimated with considerable precision (except for γ_2, they are all significantly different from zero at the 0.01 probability level). The estimated autocorrelation coefficient is significantly different from zero as well. Seasonality also turns out to have a significant effect. In fact, the value of the likelihood ratio $LR = 2[L^* - L^0]$, where L^* is the maximized value of the log-likelihood function with seasonal terms and L^0 is the maximized value of the log-likelihood function with the constraints $\gamma_{jd} = \delta_{kd} = 0$, $d = 2,3,4$, $\forall j,k$, is $LR = 187.2$. This exceeds the critical value of the corresponding χ^2 distribution with 24 degrees of freedom. Thus, the hypothesis of no seasonal effects is decisively rejected (even with the size-correction suggested by Italianer 1985).

Table 5. ML Parameter Estimates for the Stone-Geary Mixed Demand System (sample: February 1997–April 2004)

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
β_1	0.10470	0.01094	δ_{11}	-1.61772	0.23016
β_2	0.02414	0.00205	δ_{21}	-1.43719	0.16021
β_3	0.03864	0.00361	δ_{31}	-1.39137	0.18201
β_4	0.04159	0.00421	δ_{41}	-1.96160	0.24804
β_5	0.11410	0.01616	δ_{51}	-2.14579	0.51695
β_6	0.46584	0.02895	δ_{61}	-3.73940	0.56327
β_7	0.07503	0.00867	δ_{71}	-4.76009	0.67360
α_1	0.07060	0.00748	γ_{11}	-0.06237	0.02001
α_2	0.06537	0.00686	γ_{21}	0.01137	0.01049
ρ	0.43194	0.03739			

Note: Log-likelihood = 2,808.7.

Table 6 reports individual equation statistics: the $\bar{R}^2$ measure of fit and the Durbin-Watson statistics (DW) as a measure of serial correlation. Despite the extremely parsimonious parameterization implied by the Stone-Geary formulation, the fit of all equations is quite good. The DW statistics suggest that the simple parameterization chosen (a common autocorrelation coefficient for all equations) is actually fairly successful at correcting for the serial correlation of the error terms.

Table 6. Estimation Results, Summary Statistics

Share equation	$\bar{R}^2$	DW
1. Tomatoes	0.96	1.83
2. Eggplants	0.98	2.24
3. Zucchini	0.84	1.63
4. Bell peppers	0.97	2.23
5. Lettuce	0.65	1.64
6. Other vegetables	0.99	2.16
7. Legumes	0.99	2.45
8. Frozen vegetables	0.75	1.90
9. Canned vegetables	0.60	1.38

Test of Structural Change

A lively debate has recently emerged in Europe, and especially in Italy, on the inflationary effects of the introduction of the euro in January 2002. The question is whether the "changeover" was accompanied by an unexpected increase in prices, imperfectly measured by official statistics, or whether consumers' perception of the price increases were, simply put, at variance with the facts (e.g., Marini, Piergallini, and Scaramozzino (2004). The sample period of our data encompasses the date of the introduction of the euro, and the price behavior of the goods in our bundles does show some upward trend following the introduction of the euro, as illustrated in Table 7.

Because our model is largely a price-determination model (for seven of the nine goods the assumption is that supplies in any given month are given, and the corresponding prices adjust to clear the market), the hypothesis of an inflationary effect of the euro introduction suggests that we test whether our model supports a structural change occurring with the introduction of the euro. Specifically, we focus on a possible structural break occurring in January 2002, when the European common currency (the euro) replaced the lira. The specific test that we consider is based on the statistic

$$\Lambda = 2\left[\frac{T}{T_1}L_1^* + \frac{1}{2}\log\frac{T}{T_1} - L^*\right], \tag{28}$$

Table 7. Average Prices Before and After the Euro (euro/kg)

Price of:	Before the Euro January 1997–December 2001	After the Euro January 2002–April 2004
1. Tomatoes	1.45	1.81
2. Eggplants	1.25	1.48
3. Zucchini	1.49	1.84
4. Bell peppers	1.66	1.91
5. Lettuce	1.53	1.83
6. Other vegetables	1.23	1.46
7. Legumes	2.02	2.31
8. Frozen vegetables	3.13	3.35
9. Canned vegetables	0.85	1.02

where $T = 87$ and $T_1 = 59$ are the sizes of the full sample and of the sub-sample up to the hypothesized structural break (i.e., February 1997 to December 2001), L^* is the maximized value of the log-likelihood function over the entire sample, and L_1^* is the maximized value of the log-likelihood function over the sub-sample of T_1. We find that the computed value of this statistic is $\Lambda = 128.3$. Under the null hypothesis of parameter stability, the statistic Λ is distributed as χ^2 with $(T - T_1)N = 224$ degrees of freedom, where $N = 8$ is the number of estimating equations (Anderson and Blundell 1984). Thus, we find no structural change: the hypothesis of constancy of the parameters after the introduction of the euro is not rejected at the customary 0.05 significance level.

Elasticities

As with standard demand analysis, the sensitivity of the endogenous variables to changes in predetermined variables is best illustrated using elasticities. Table 8 reports the estimated Hicksian elasticities derived from our mixed demand equations, evaluated at the sample mean of the predetermined variables ($p_i = z_k = y = 1$). Given that the estimated coefficients satisfy the Stone-Geary regularity conditions, the own-quantity and own-price effects are negative. The substitution pattern displayed by the cross-price elasticities are heavily constrained by the fact that, as noted earlier, with the Stone-Geary utility function all goods are *R*-substitutes in Madden’s (1991) sense. These elasticities, of course, also satisfy additional restrictions because of the homogeneity and adding-up properties—see Moschini and Vissa (1993) for an explicit statement of such restrictions.

Table 8. Hicksian Mixed Demand Elasticities (Evaluated at the Sample Mean)

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	p_1	p_2
q_1	-0.68	-0.07	-0.12	-0.10	-0.27	-0.72	-0.10	0.52	0.48
q_2	-0.29	-0.48	-0.12	-0.10	-0.27	-0.72	-0.10	0.52	0.48
q_3	-0.29	-0.07	-0.54	-0.10	-0.27	-0.72	-0.10	0.52	0.48
q_4	-0.29	-0.07	-0.12	-0.44	-0.27	-0.72	-0.10	0.52	0.48
q_5	-0.29	-0.07	-0.12	-0.10	-0.59	-0.72	-0.10	0.52	0.48
q_6	-0.29	-0.07	-0.12	-0.10	-0.27	-0.93	-0.10	0.52	0.48
q_7	-0.29	-0.07	-0.12	-0.10	-0.27	-0.72	-0.27	0.52	0.48
x_1	-0.42	-0.11	-0.17	-0.15	-0.38	-1.04	-0.14	-0.69	0.69
x_2	-0.28	-0.07	-0.11	-0.10	-0.25	-0.68	-0.09	0.49	-0.49

Note: Each entry is the elasticity of the row variable with respect to the column variable. Indices for (q,z) variables: 1 = tomatoes, 2 = eggplants, 3 = zucchini, 4 = bell peppers, 5 = lettuce, 6 = other fresh vegetables, 7 = legumes. Indices for (p,x) variables: 1 = frozen vegetables, 2 = canned vegetables.

Perhaps of more immediate interest are the Marshallian elasticities. Rather than reporting such elasticities for the estimated mixed demand system, we have computed the Marshallian elasticities of the implied direct demand equations. Moschini and Vissa (1993) discuss the general method for reconstructing elasticities of ordinary demand functions from estimated mixed demand functions. Here, however, our task is simplified because the assumed preferences admit an explicit solution for Marshallian direct demand equations, as given in equations (18)–(19). The price and income elasticities of these demand equations are reported in Table 9.

Expenditure elasticities, reported in the last column, suggest that non-fresh products have a more inelastic demand than fresh vegetable products (that is, the latter have more of the "luxury" good attribute). Own-price elasticities are all negative, as expected (all goods must be normal with Stone-Geary preferences, and thus Giffen goods are ruled out). Also of interest is that, with the exception of canned vegetables, all products display an elastic demand. Indeed, the own-price elasticity values indicate that some of the products are very elastic (legumes, for example, have an own-price elasticity of -5.40). Thus, we confirm Moschini and Vissa's (1993) observation that mixed demand estimation seems to yield more elastic demand relations than direct demand estimation (not surprisingly, perhaps, given the differing assumptions about the source of the error). In any event, we should note that the Marshallian elasticities in Table 9 are conditional on aggregate expenditures on vegetables and should be interpreted accordingly.

Table 9. Marshallian Direct Demand Elasticities Implied by Mixed Demand Estimates (evaluated at the sample mean)

	q_1	q_2	q_3	q_4	q_5	q_6	q_7	p_1	p_2	y
z_1	-2.45	0.04	0.06	0.07	0.20	0.96	0.16	0.06	-0.01	0.91
z_2	0.16	-2.40	0.05	0.07	0.19	0.90	0.15	0.05	-0.01	0.84
z_3	0.15	0.03	-2.34	0.07	0.19	0.88	0.15	0.05	-0.01	0.83
z_4	0.19	0.04	0.07	-2.88	0.23	1.09	0.18	0.06	-0.01	1.02
z_5	0.20	0.04	0.07	0.09	-2.90	1.16	0.20	0.07	-0.01	1.09
z_6	0.31	0.07	0.11	0.13	0.37	-3.00	0.29	0.10	-0.02	1.64
z_7	0.37	0.08	0.13	0.16	0.45	2.12	-5.40	0.12	-0.02	1.99
x_1	0.09	0.02	0.03	0.04	0.11	0.53	0.09	-1.41	-0.01	0.50
x_2	0.06	0.01	0.02	0.03	0.07	0.35	0.06	0.02	-0.95	0.33

Note: Each entry is the elasticity of the row variable with respect to the column variable. Indices for (q,z) variables: 1 = tomatoes, 2 = eggplants, 3 = zucchini, 4 = bell peppers, 5 = lettuce, 6 = other fresh vegetables, 7 = legumes. Indices for (p,x) variables: 1 = frozen vegetables, 2 = canned vegetables.

Conclusions

IN THIS CHAPTER WE HAVE REVISITED the idea of approaching empirical demand analysis with a mixed demand system. After briefly motivating the study, and reviewing the relevant theory, we have analyzed in detail the mixed demand system that one can obtain from the Stone-Geary utility function, the preference relation underlying the standard LES model. These preferences, of course, are known to be restrictive. But because it has not been presented before, the mixed demand system that we have derived should be of interest, as a benchmark at a minimum. In fact, the properties of aggregation across consumers enjoyed by Stone-Geary preferences (a special case of quasi-homothetic preferences), and the parsimonious nature of the parameterization, may still make this specific model of interest, in its own right, for large-scale demand systems. In addition, because the Stone-Geary preferences allow for the explicit solution of both direct and mixed demand functions, the model that we have detailed is useful as a framework with which to explore some of the implications of the stochastic specification of demand systems. We have illustrated this last attribute with a simple Monte Carlo experiment. The results of our simulations confirm the importance of making correct assumptions about which variables to take as predetermined in empirical demand models and thus vindicate our presumption that mixed demand models deserve more attention.

The system developed was illustrated with an application to the Italian demand for vegetables. Specifically, we estimated a nine-good mixed demand system for the group of fresh and processed vegetables, whereby we allow for seven goods (categories of fresh vegetables) to be represented by predetermined supply, with price adjusting to clear the market, and for two goods (canned and frozen vegetables) to have the standard representation (price is given and quantity adjusts). The system was estimated with monthly data obtained from a large representative and stratified sample of Italian households. This application illustrates that the new system that we have derived is readily estimable and can be quite useful when a mixed demand model is called for. The fit of the estimated equations is good and the one-parameter correction for serial correlation seems to work reasonably well. The estimated elasticities appear plausible, although more work needs to be done to characterize more carefully the substitutability patterns (inevitably constrained by our restrictive preference assumptions).

References

Anderson, G., and R. Blundell. 1984. "Consumer Non-Durables in the UK: A Dynamic Demand System." *Economic Journal Conference Papers* 94: 34–44.

Barten, A.P. 1992. "The Estimation of Mixed Demand Systems." In R. Bewley and Tran Van Hoa, eds., *Contributions to Consumer Demand and Econometrics: Essays in Honour of Henri Theil*. London: Macmillan.

Barten, A.P., and L.J. Bettendorf. 1989. "Price Formation of Fish: An Application of an Inverse Demand System." *European Economic Review* 33(8): 1509–1525.

Berndt, E.R., and N.E. Savin. 1975. "Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances." *Econometrica* 43(5–6): 937–957.

Blackorby, C., D. Primont, and R.R. Russell. 1978. *Duality, Separability, and Functional Structure: Theory and Economic Applications.* New York: North-Holland.

Chavas, J-P. 1984. "The Theory of Mixed Demand Functions." *European Economic Review* 24(3): 321–344.

Christensen, L.R., D.W. Jorgenson, and L.J. Lau. 1975. "Transcendental Logarithmic Utility Functions." *American Economic Review* 65(3): 367–383.

Cunha-e-Sá, M.A., and M.M. Ducla-Soares. 1999. "Specification Tests for Mixed Demand Systems with an Emphasis on Combining Contingent Valuation and Revealed Data." *Journal of Environmental Economics and Management* 38(2): 215–233.

Davidson, R., and J.G. MacKinnon. 1993. *Estimation and Inference in Econometrics.* Oxford: Oxford University Press.

Deaton, A. 1981. "Theoretical and Empirical Approaches to Consumer Demand Under Rationing." In A. Deaton, ed., *Essays in the Theory and Measurement of Consumer Behaviour in Honour of Sir Richard Stone*. Cambridge: Cambridge University Press.

Deaton, A., and J. Muellbauer. 1980a. *Economics and Consumer Behavio*r. Cambridge: Cambridge University Press.

______. 1980b. "An Almost Ideal Demand System." *American Economic Review* 70(3): 312–326.

Fuller, W.A. 1987. *Measurement Error Models*. New York: Wiley.

Geary, R.C. 1949–50. "A Note on 'A Constant Utility Index of the Cost of Living'." *Review of Economic Studies* 18: 65–66.

Gorman, W.M. 1976. "Tricks with Utility Functions." In M. Artis and R. Nobay, eds., *Essays in Economic Analysis*. Cambridge: Cambridge University Press.

Heien, D.M. 1977. "Price Determination Processes for Agricultural Sector Models." *American Journal of Agricultural Economics* 59(1): 126–132.

Holt, M.T., and R.C. Bishop. 2002. "A Semiflexible Normalized Quadratic Inverse Demand System: An Application to the Price Formation of Fish." *Empirical Economics* 27(1): 23–47.

Italianer, A. 1985. "A Small-Sample Correction for the Likelihood Ratio Test." *Economics Letters* 19(4): 315–317.

Johnson, S.R., Z.A. Hassan, and R.D. Green. 1984. *Demand System Estimation: Methods and Applications*. Ames, IA: Iowa State University Press.

Klein, L.R., and H. Rubin. 1947–48. "A Constant-Utility Index of the Cost of Living." *Review of Economic Studies* 15: 84–87.

Madden, P. 1991. "A Generalization of Hicksian q Substitutes and Complements with Application to Demand Rationing." *Econometrica* 59(5): 1497–1508.

Marini, G., A. Piergallini, and P. Scaramozzino. 2004. "Inflation Bias after the Euro: Evidence from the UK and Italy." Working Paper, Department of Economics, University of Rome "Tor Vergata," August.

Matsuda, T. 2004. "Incorporating Generalized Marginal Budget Shares in a Mixed Demand System." *American Journal of Agricultural Economics* 86(4): 1117–1126.

Moschini, G. 1998. "The Semiflexible Almost Ideal Demand System." *European Economic Review* 42(2): 349–364.

______. 2001. "Production Risk and the Estimation of Ex-ante Cost Functions." *Journal of Econometrics* 100(2): 357–380.

Moschini, G., and D. Moro. 1994. "Autocorrelation Specification in Singular Equation Systems." *Economics Letters* 46(4): 303–309.

Moschini, G., and P.L. Rizzi. 2006. "The Normalized Quadratic Mixed Demand System." Unpublished manuscript, Department of Economics, Iowa State University.

Moschini, G., and A. Vissa. 1992. "A Linear Inverse Demand System." *Journal of Agricultural and Resource Economics* 17(2): 294–302.

______. 1993. "Flexible Specification of Mixed Demand Systems." *American Journal of Agricultural Economics* 75(1): 1–9.

Neary, J.P. 1997. "R.C. Geary's Contributions to Economic Theory." In D. Conniffe, ed., *Roy Geary, 1896-1983: Irish Statistician* (Centenary Lecture by John E. Spencer and Associated Papers). Dublin: Oak Tree Press.

Neary, J.P., and K.W.S. Roberts. 1980. "The Theory of Household Behavior Under Rationing." *European Economic Review* 13(1): 25–42.

Samuelson, P.A. 1947–48. "Some Implications of 'Linearity'." *Review of Economic Studies* 15: 88–90.

______. 1965. "Using Full Duality to Show That Simultaneously Additive Direct and Indirect Utilities Implies Unitary Price Elasticity of Demand." *Econometrica* 33(4): 781–796.

Stone, J.R.N. 1954. "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand." *Economic Journal* 54: 511–527.

Theil, H. 1975 and 1976. *The Theory and Measurement of Consumer Demand* (Vols. I and II). Amsterdam: North-Holland.

Appendix

TO SOLVE THE FIRST-ORDER CONDITIONS in (10)–(12), note from (10) that the solutions for variables x_j would be readily obtained upon having solved for the Lagrange multiplier λ. Thus, substituting the n equations (10) into the budget constraint (12), and rearranging (11), the m + 1 equations to be solved are

$$\lambda \sum_{k=1}^{m} q_k z_k = \lambda\left(y - \sum_{i=1}^{n} p_i \gamma_i \right) - \sum_{i=1}^{n} \alpha_i \tag{A1}$$

$$\frac{\delta_s q_s}{\left(y - \sum_{i=1}^{n} p_i \gamma_i - \sum_{k=1}^{m} q_k \delta_k \right)} + \beta_s = \lambda q_s z_s \qquad s = 1,\ldots,m \,. \tag{A2}$$

As it stands, the system is nonlinear in the variables of interest λ and q_s ($s = 1,\ldots m$), but it can be made linear in transformed variables as follows. Sum the m equations in (A2), use the budget constraint in (A1), and recall that $\sum_{s=1}^{m} \beta_s = 1 - \sum_{j=1}^{n} \alpha_j$, to obtain

$$\lambda = \frac{1}{\left(y - \sum_{i=1}^{n} p_i \gamma_i - \sum_{k=1}^{m} q_k \delta_k \right)} \,. \tag{A3}$$

By using (A3) in (A2), it follows that the system of m + 1 equations to be solved is reduced to

$$\lambda q_s \delta_s + \beta_s = \lambda q_s z_s \qquad s = 1,\ldots,m \tag{A4}$$

$$\lambda \sum_{k=1}^{m} q_k z_k = \lambda\left(y - \sum_{i=1}^{n} p_i \gamma_i \right) - \sum_{i=1}^{n} \alpha_i \,. \tag{A5}$$

This system of equations is linear in the transformed variables $L \equiv \lambda(y - \sum_{i=1}^{n} p_i \gamma_i)$ and $Q_s \equiv \lambda q_s (s = 1,\ldots,m)$. Solving these linear equations, and re-expressing the results in terms of the variables of interest, yields the mixed demand equations in (13)–(14).